X-logics based multivalued reasoning for dialogical agents (ongoing work)

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  - aim to achieve reasoning on arguments;
- One of our far(!) ideal(!!) goal : simulation of strategies for the 'game' of argumentation

Outline

1. X–logics
2. Their use in the context of a dialogical framework
3. Nmatrices, Nsequents
4. Transformation into classical sequents
5. LA, logic of attitudes
6. Links with $MSPL$ (Avron et Al.)
7. Epilog

Aim : attempt for defining a dialogical framework in which two 'agents' can achieve 'some' reasoning on their arguments.

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Theorem

$$K \vdash_X f \quad iff \quad \overline{K \cup \{f\}} \cap X \subseteq \overline{K}$$
$$i.e. \quad (\forall x \in X)(K \wedge f \vdash x \Rightarrow K \vdash x)$$

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Vocabulary

f is *compatible* with K regarding X iff $K \vdash_X f$, *incompatible* otherwise.

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- $\{a\} \vdash_{\{\bot\}} b$ *and* $\{a, \neg b\} \nvdash_{\{\bot\}} b$

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- $\{b\} \vdash_{\{\bot\}} a \wedge b$ *and* $\{b\} \vdash_{\{\bot\}} \neg(a \wedge b)$

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- $\{a\} \vdash_{\{\bot\}} b \quad and \quad \{a, \neg b\} \not\vdash_{\{\bot\}} b$
- $\{b\} \vdash_{\{\bot\}} a \wedge b \quad and \quad \{b\} \vdash_{\{\bot\}} \neg(a \wedge b)$
- $\{b\} \not\vdash_{\{\bot,a,\neg a\}} a \wedge b \quad and \quad \{b\} \not\vdash_{\{\bot,a,\neg a\}} \neg(a \wedge b)$

Why X-logics for argumentation?

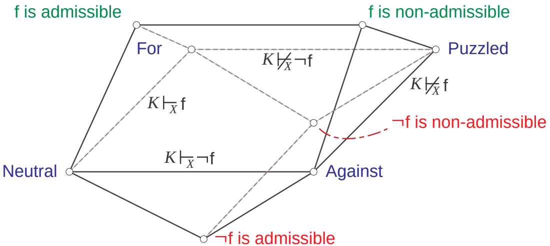
- they correspond to *permissive* inference relations (Bochman);
- as such, they caracterize a broad family of *supra-classical* relations;
- travel among relations *via* $2^{\mathcal{L}}$;
- the inner structure of X allows to construct different kind of logics, hence different kinds of agents;
- provide parts of the underlying langage with a “logical” status;
- also(!): among the different Xs, try to get fragments of lower complexity...

Agents and attitudes

Definition

- *An* agent *is a couple* $[K, X]$, *with* K *a consistant set of formulas, and* X *a set of formulas containing* $\perp$. *The set of all agents is written* $\mathcal{A}$.
- *a formula* f *is* admissible *by an agent* $[K, X]$ *iff this formula is compatible with* K *regarding* X.

Attitudes



Answers and arguments

Definition

- *An answer of the agent* $\Phi = [K, X]$ *to a set* F *is a set* A *composed both of formulas of* K *and of negated formulas of* X*, and such that* F *is incompatible with* K *regarding* X*.*
- *An argument* α *given by the agent* $[K, X]$ *in the presence of a formula* C *is a couple* $\langle S, \neg C \rangle$ *such that* S *is an answer to* C*.* S *and* $\neg C$ *are respectively the support and the conclusion of the argument.*

Definition

Given α *et* β *two arguments, and* $\{s_1, \ldots, s_n\} \subseteq supp(\beta)$ *:*

- α *attacks* β *iff* $concl(\alpha) = \neg(s_1 \wedge \cdots \wedge s_n)$
- α *defends* β *iff* $concl(\alpha) = s_1 \wedge \cdots \wedge s_n$

Example: the deafs dialogue

1: "You are *rigid*, be *flexible*"
2: "No. YOU are *lax*, be *thorough*"

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- Consider $[K_1, X_1]$ and $[K_2, X_2]$ with

$$\begin{aligned} K_1 &= \{Flexible \Rightarrow \neg Lax, \neg Rigid \Rightarrow Flexible\} \\ X_1 &= \{Rigid\} \\ K_2 &= \{Thorough \Rightarrow \neg Rigid, \neg Lax \Rightarrow Thorough\} \\ X_2 &= \{Lax\} \end{aligned}$$

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- Answer of 2 to *Rigid* is $A = \{\neg Lax\} \cup K_2$ (with $K_2 \not\vdash_{\{\bot, Lax\}} Rigid$)
- Counter-argument from 2 against *Rigid*: $\langle A, \neg Rigid\rangle$, which deductively amounts to: $\langle A, Thorough\rangle$

Arguments and attitudes

Properties

If $[K, X]$ is

- *against* a subset of $supp(\alpha)$, it can construct at least one argument attacking α;
- *for* a subset of $supp(\alpha)$, it can construct at least one argument defending α;
- *puzzled* by a subset of $supp(\alpha)$, it can construct at least one argument both attacking and defending α;
- *neutral* about a subset of the support of $supp(\alpha)$, then it has no argument to give about α.

Reasoning about attitudes...

Properties

- $[K, X]$ is *for* f iff it is *against* $\neg f$,
- $[K, X]$ is *neutral* about f iff it is *neutral* about $\neg f$,
- $[K, X]$ is *puzzled* about f iff it is *puzzled* about $\neg f$,
- $[K, X]$ is *for* the tautologies,
- $[K, X]$ is *against* the contradictions.

If (for instance) $[K, X]$ is *for* f, and *for* g, which attitude will it be able to adopt regarding f and g, f or $g \ldots$?

Example: combining attitudes

- Consider $[K, X]$, with

$$
\begin{aligned}
K &= \{Inflation \Rightarrow \neg IncreasingPurchasingPower, \\
&\quad IncreasingSalaries \Rightarrow IncreasingPurchasingPower, \\
&\quad FixingBasicPrices \Rightarrow IncreasinPurchasingPower, \\
&\quad IncreasingSalaries \wedge FixingBasicPrices \Rightarrow Inflation\} \\
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- K is both *for IncreasingSalaries* and *for FixingBasicPrices*
- K is *against* $IncreasingSalaries \wedge FixingBasicPrices$

Valuation

Consider the valuation

$$v_4^{\vdash x} : \mathcal{A} \times \mathcal{P} \longrightarrow \mathcal{FOUR}$$

such that $\forall A \in \mathcal{A}, \forall p \in \mathcal{P}$,

$$\begin{array}{lll} v_4^{\vdash x}(A,p) = 1 & \text{iff} & A \text{ is } \textit{for } p \\ v_4^{\vdash x}(A,p) = \top & \text{iff} & A \text{ is } \textit{neutral} \text{ regarding } p \\ v_4^{\vdash x}(A,p) = \bot & \text{iff} & A \text{ is } \textit{puzzled} \text{ regarding } p \\ v_4^{\vdash x}(A,p) = 0 & \text{iff} & A \text{ is } \textit{against } p \end{array}$$

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1. adding constraints to X in order to determinize in an unique way the admissibility associated with the different logical combinations of two formulas;
2. extending our valuation to *sets* of truth values
   $\rightarrow$ non-deterministic multi-valued logics

Nmatrices [Avron et Al.]

The *Nmatrice* $\mathcal{M}_{LA}$ associated to $\mathcal{FOUR}$ is a triple $\mathcal{M} = \langle \mathcal{V}, \mathcal{D}, \mathcal{O} \rangle$ with

1. $\mathcal{V} = \{O, \bot, \top, 1\}$, set of truth values;
2. $\mathcal{D} = \{1, \top\}$, set of designated valued,
3. *Non-designated* values: $\mathcal{N} = \mathcal{V} \setminus \mathcal{D}$;
4. $\mathcal{O} = \{\neg, \vee, \wedge\}$, set of operators whose behaviour is described by the corresponding truth tables:

α	$\neg\alpha$
1	$\{0\}$
$\top$	$\{\top\}$
$\bot$	$\{\bot\}$
0	$\{1\}$

$\alpha \wedge \beta$	1	$\top$	$\bot$	0
1	$\{1, \top, \bot, 0\}$	$\{\top, 0\}$	$\{\bot, 0\}$	$\{0\}$
$\top$	$\{\top, 0\}$	$\{\top, 0\}$	$\{0\}$	$\{0\}$
$\bot$	$\{\bot, 0\}$	$\{0\}$	$\{\bot, 0\}$	$\{0\}$
0	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$

$\alpha \vee \beta$	1	$\top$	$\bot$	0
1	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$
$\top$	$\{1\}$	$\{1, \top\}$	$\{1\}$	$\{1, \top\}$
$\bot$	$\{1\}$	$\{1\}$	$\{1, \bot\}$	$\{\bot, 1\}$
0	$\{1\}$	$\{1, \top\}$	$\{1, \bot\}$	$\{1, \top, \bot, 0\}$

Nsequent

Multivalued Sequent

- *Sequent* in a matrix $\mathcal{M}$ = set of signed formulas
- The classical sequent $\Gamma \Rightarrow \Delta$ is interpreted by $\{0 : \Gamma\} \cup \{1 : \Delta\}$ où $\mathcal{V} = \{0, 1\}$ et $\mathcal{D} = \{1\}$
- Conventions : $\mathcal{V} = \{t_0, \ldots, t_{n-1}\}$ (with $n \geq 2$) and $\mathcal{D} = \{t_d, \ldots, t_{n-1}\}$ (with $d \geq 1$)

Definition

A n–sequent on a language $\mathcal{L}$ is an expression Σ of the form $\Gamma_0|\Gamma_1|\ldots|\Gamma_{n-1}$ where for every $0 \leq i \leq n-1$, Γ_i is a finite set of formulas on $\mathcal{L}$.

Notation

Replace | by $\Rightarrow$: $\Gamma_{i_1}|\ldots\Gamma_{i_r} \Rightarrow \Gamma_{j_1}|\ldots|\Gamma_{j_s}$ where $i_1, \ldots, i_r \in \mathcal{N}$ and $j_1, \ldots, j_s \in \mathcal{D}$

Multivalued sequents for *LA*

- Axioms: any set of signed formulas of the form $\{a : \varphi \mid a \in \mathcal{V}, \varphi \in \mathcal{F}\}$
- Structural rules: weakening
- Logical rules (after simplification), e.g.:

Conjonction:

$$\frac{\Omega,\bot{:}\varphi,1{:}\varphi \quad \Omega,\bot{:}\psi,1{:}\psi \quad \Omega,\bot{:}\varphi,\psi}{\Omega,\bot{:}\varphi\wedge\psi,0{:}\varphi\wedge\psi}$$

$$\frac{\Omega,1{:}\varphi,\top{:}\varphi \quad \Omega,1{:}\psi,\top{:}\psi \quad \Omega,\top{:}\varphi,\psi}{\Omega,0{:}\varphi\wedge\psi,\top{:}\varphi\wedge\psi}$$

$$\frac{\Omega,\bot{:}\varphi,\psi,0{:}\varphi,\psi \quad \Omega,0{:}\varphi,\psi,\top{:}\varphi,\psi}{\Omega,0{:}\varphi\wedge\psi}$$

Transformation into Nsequents

Example:

Conjonction:

$$\frac{\Gamma_\bot,\varphi|\Gamma_0\Rightarrow\Gamma_1,\varphi|\Gamma_\top \quad \Gamma_\bot,\psi|\Gamma_0\Rightarrow\Gamma_1,\psi|\Gamma_\top \quad \Gamma_\bot,\varphi,\psi|\Gamma_0\Rightarrow\Gamma_1|\Gamma_\top}{\Gamma_\bot,\varphi\wedge\psi|\Gamma_0,\varphi\wedge\psi\Rightarrow\Gamma_1|\Gamma_\top}$$

$$\frac{\Gamma_\bot|\Gamma_0\Rightarrow\Gamma_1,\varphi|\Gamma_\top,\varphi \quad \Gamma_\bot|\Gamma_0\Rightarrow\Gamma_1,\psi|\Gamma_\top,\psi \quad \Gamma_\bot|\Gamma_0\Rightarrow\Gamma_1|\Gamma_\top,\varphi,\psi}{\Gamma_\bot|\Gamma_0,\varphi\wedge\psi\Rightarrow\Gamma_1|\Gamma_\top,\varphi\wedge\psi}$$

$$\frac{\Gamma_\bot,\varphi,\psi|\Gamma_0,\varphi,\psi\Rightarrow\Gamma_1|\Gamma_\top \quad \Gamma_\bot|\Gamma_0,\varphi,\psi\Rightarrow\Gamma_1|\Gamma_\top,\varphi,\psi}{\Gamma_\bot|\Gamma_0,\varphi\wedge\psi\Rightarrow\Gamma_1|\Gamma_\top}$$

Expressiveness condition

- Condition(Avron, Ben–naim, Konikowska, 07): a *n*–sequent calculus can be translated into a two–sided sequent calculus only if the underlying langage is *sufficiently expressive* for the semantics induced by the Nmatrix $\mathcal{M}$
- Intuition: for any valuation of an initial formula, by introducing new formulas compounded only from the initial formula with any connector, one can still adress any subsequent valuation of these new formulas either in $\mathcal{N}$ or $\mathcal{D}$
- This ensures (in a strong combinatoric way) the partition of any multi-valued sequent into a two-valued sequent

partition sequence

$\Sigma = \Gamma_1 \mid \Gamma_2 \mid \ldots \mid \Gamma_n$ a n–sequent de $\mathcal{L}$.

Definition (Avron, Ben–naim, Konikowska, 07)

A partition sequence for Σ *is a tuple* $\pi = \langle \pi_1, \ldots, \pi_n \rangle$ *such that for* $1 \leq i \leq n$, π_i *is a partition of* Γ_i *of the form*

$$\pi_i = \{\Gamma'_{ij} \mid 1 \leq j \leq l_i\} \cup \{\Gamma''_{ik} \mid 1 \leq k \leq m_i\}$$

Nsequents and classical sequents

For a partition sequence π and for all $1 \leq i \leq n$, define:

$$\Delta_i^{'} = \bigcup \{A_j^i(\Gamma_{ij}^{'}) \mid 1 \leq j \leq l_i\}$$

$$\Delta_i^{''} = \bigcup \{B_k^i(\Gamma_{ik}^{''}) \mid 1 \leq k \leq m_i\}$$

$$\Sigma_\pi = \Delta_1^{'}, \Delta_2^{'}, \ldots, \Delta_n^{'} \Rightarrow \Delta_1^{''}, \Delta_2^{''}, \ldots, \Delta_n^{''}$$

where $A_j^i(\Gamma_{ij}^{'}) = \{A_j^i\varphi \mid \varphi \in \Gamma_{ij}^{'}\}$ and $B_k^i(\Gamma_{ik}^{''})$ is defined in the same way. Let Π be the set of all partition sequences of Σ, the *set of two-sided sequents generated by* $TWO(\Sigma) = \{\Sigma_\pi \mid \pi \in \Pi\}$

Theorem (Avron, Ben–naim, Konikowska, 07)

If $\mathcal{L}$ is sufficiently expressive langage for every n-sequent $\Sigma = \Gamma_1 \mid \Gamma_2 \mid \ldots \mid \Gamma_n$, and any valuation v of formulas of $\mathcal{L}$, $v \models \Sigma$ iff $v \models \Sigma'$ for every $\Sigma' \in TWO(\Sigma)$.

Preliminary results

Theorem

For every agent A and every formula α:

$v_4^{\vdash x}(A,\alpha) = 1$ *iff* $v_4^{\vdash x}(A,\alpha) \in \mathcal{D}$ *and* $v_4^{\vdash x}(A,\neg\alpha) \in \mathcal{N}$

$v_4^{\vdash x}(A,\alpha) = \top$ *iff* $v_4^{\vdash x}(A,\alpha) \in \mathcal{D}$ *and* $v_4^{\vdash x}(A,\neg\alpha) \in \mathcal{D}$

$v_4^{\vdash x}(A,\alpha) = \bot$ *iff* $v_4^{\vdash x}(A,\alpha) \in \mathcal{N}$ *and* $v_4^{\vdash x}(A,\neg\alpha) \in \mathcal{N}$

$v_4^{\vdash x}(A,\alpha) = 0$ *iff* $v_4^{\vdash x}(A,\alpha) \in \mathcal{N}$ *and* $v_4^{\vdash x}(A,\neg\alpha) \in \mathcal{D}$

$\rightarrow$ ensures the two-sided partition of every sequent $\Sigma = \Gamma_1 \mid \Gamma_2 \mid \ldots \mid \Gamma_n$

Calculus $\mathcal{S}_{LA}$

Axioms: $\varphi \Rightarrow \varphi$ for every formula φ

Rules:

$$\frac{\Gamma\Rightarrow\Delta,\varphi,\psi}{\Gamma\Rightarrow\Delta,\varphi\vee\psi} \qquad \frac{\Gamma,\varphi,\psi\Rightarrow\Delta}{\Gamma,\varphi\wedge\psi\Rightarrow\Delta}$$

$$\frac{\Gamma,\neg\varphi,\neg\psi\Rightarrow\Delta}{\Gamma,\neg(\varphi\vee\psi)\Rightarrow\Delta} \qquad \frac{\Gamma\Rightarrow\Delta,\neg\varphi,\neg\psi}{\Gamma\Rightarrow\Delta,\neg(\varphi\wedge\psi)}$$

$$\frac{\Gamma\Rightarrow\Delta,\varphi}{\Gamma\Rightarrow\Delta,\neg\neg\varphi} \qquad \frac{\Gamma,\varphi\Rightarrow\Delta}{\Gamma,\neg\neg\varphi\Rightarrow\Delta}$$

- $\mathcal{S}_{LA}$ gets only one of the two rules for disjonction and only one of the two rules for conjonction
- None of the classical rules for negation
- The calculus is symetrical

MSPL (Avron, Ben–naim, Konikowska, 07)

Given a set S of sources of information and a processor P

- each source $s \in S$ can tell if a formula ϕ is true, if it is false or if it has no information about ϕ
- The processor P collects the formulas and combines them from the informations given by the sources :
  - it has information that ϕ is true but no information that ϕ is false
  - it has information that ϕ is false but no information that ϕ is true
  - it has both informations that ϕ is true and that ϕ is false
  - it has no information on ϕ at all

MSPL (Avron, Ben–naim, Konikowska, 07)

The *Nmatrice MSPL* associated to $\mathcal{FOUR}$ is $\mathcal{M} = \langle \mathcal{V}, \mathcal{D}, \mathcal{O} \rangle$ with

1. $\mathcal{V} = \{O, \bot, \top, 1\}$, set of truth values;
2. $\mathcal{D} = \{1, \top\}$, the designated values,
3. $\mathcal{N} = \mathcal{V} \setminus \mathcal{D}$;
4. $\mathcal{O} = \{\neg, \vee, \wedge\}$ the set of operators described by the following truth-tables:

α	$\neg\alpha$
1	$\{0\}$
$\top$	$\{\top\}$
$\bot$	$\{\bot\}$
0	$\{1\}$

$\alpha \wedge \beta$	1	$\top$	$\bot$	0
1	$\{1, \top\}$	$\{\top\}$	$\{\bot, 0\}$	$\{0\}$
$\top$	$\{\top\}$	$\{\top\}$	$\{0\}$	$\{0\}$
$\bot$	$\{\bot, 0\}$	$\{0\}$	$\{\bot, 0\}$	$\{0\}$
0	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$

$\alpha \vee \beta$	1	$\top$	$\bot$	0
1	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$
$\top$	$\{1\}$	$\{\top\}$	$\{1\}$	$\{\top\}$
$\bot$	$\{1\}$	$\{1\}$	$\{1, \bot\}$	$\{\bot, 1\}$
0	$\{1\}$	$\{\top\}$	$\{1, \bot\}$	$\{\top, 0\}$

LA vs. MSPL

LA

$$\frac{\Gamma \Rightarrow \Delta, \varphi, \psi}{\Gamma \Rightarrow \Delta, \varphi \vee \psi} \qquad \frac{\Gamma, \varphi, \psi \Rightarrow \Delta}{\Gamma, \varphi \wedge \psi \Rightarrow \Delta}$$

$$\frac{\Gamma, \neg\varphi, \neg\psi \Rightarrow \Delta}{\Gamma, \neg(\varphi \vee \psi) \Rightarrow \Delta} \qquad \frac{\Gamma \Rightarrow \Delta, \neg\varphi, \neg\psi}{\Gamma \Rightarrow \Delta, \neg(\varphi \wedge \psi)}$$

$$\frac{\Gamma \Rightarrow \Delta, \varphi}{\Gamma \Rightarrow \Delta, \neg\neg\varphi} \qquad \frac{\Gamma, \varphi \Rightarrow \Delta}{\Gamma, \neg\neg\varphi \Rightarrow \Delta}$$

LA vs. MSPL

LA

$$\frac{\Gamma \Rightarrow \Delta, \varphi, \psi}{\Gamma \Rightarrow \Delta, \varphi \vee \psi} \qquad \frac{\Gamma, \varphi, \psi \Rightarrow \Delta}{\Gamma, \varphi \wedge \psi \Rightarrow \Delta}$$

$$\frac{\Gamma, \neg\varphi, \neg\psi \Rightarrow \Delta}{\Gamma, \neg(\varphi \vee \psi) \Rightarrow \Delta} \qquad \frac{\Gamma \Rightarrow \Delta, \neg\varphi, \neg\psi}{\Gamma \Rightarrow \Delta, \neg(\varphi \wedge \psi)}$$

$$\frac{\Gamma \Rightarrow \Delta, \varphi}{\Gamma \Rightarrow \Delta, \neg\neg\varphi} \qquad \frac{\Gamma, \varphi \Rightarrow \Delta}{\Gamma, \neg\neg\varphi \Rightarrow \Delta}$$

MSPL

$$= LA + \quad \frac{\Gamma \Rightarrow \Delta \neg\varphi \quad \Gamma \Rightarrow \Delta \neg\psi}{\Gamma \Rightarrow \Delta, \neg(\varphi \vee \psi)} \quad \frac{\Gamma \Rightarrow \Delta \varphi \quad \Gamma \Rightarrow \Delta \psi}{\Gamma \Rightarrow \Delta, \varphi \wedge \psi}$$

Back to cumulativity

- Consider (roughly) $X \risingdotseq X'$ iff $(\vdash_X \Leftrightarrow \vdash_{X'})$ and $\hat{X}$ the least representative of the corresponding equivalence class
- Define $S^\wedge$ as the *(And)*-closure of S
- If $(\hat{X}^c)^\wedge = \hat{X}^c$ then
  - *Conjunctive Cautions Monotony* holds;
  - *LA* turns into *MSPL*.

Epilog

- A non-deterministic multivalued calculus with four truth values
- Describes how an agent evaluates a compound formula from its elementary attitudes
- A *generic* 'classical' calculus $\mathcal{S}_{LA}$
- Describes how an agent admits a compound formula from the admissibility of its subformulas
- Since $\mathcal{S}_{LA}$ relies on the only distinction between designated and non-designated values, it amounts to the common behaviour of all the agents
- horizon: investigate more accurately the role of *MSPL* in argumentation
- Far horizon: relating the reasoning of an agent with strategies of construction of new arguments
- Complementary direction: how additional constraints on X can determinize the connectors

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